MIDTERM EXAMINATION
Fall 2010

Calculus & Analytical Geometry-I

MIDTERM SOLVED PAPERS (PAPER #2)

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ..... At x_0

► Minima
► Maxima
► None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ..... At x_0

► Minima
► Maxima
► None of these

Question No: 3 (Marks: 1) - Please choose one

A line $x = x_0$ is called ---------- for the graph of a function f if $f(x) \to +\infty \text{ or } f(x) \to -\infty$ as x approaches x_0 from the right or from the left

► Horizontal asymptotes
► None of these
► Vertical asymptotes

Question No: 4 (Marks: 1) - Please choose one

A line $y = y_0$ is called a ..... for the graph f if $\lim_{x \to +\infty} f(x) = y_0 \text{ or } \lim_{x \to -\infty} f(x) = y_0$

► Vertical asymptotes
► Horizontal asymptotes

- None of these

Question No: 5 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then

$\frac{d}{dx}[x^n] =$

- x^{n-1}
- $n x^{n-1}$
- $n x^{n+1}$
- $(n-1)x^{n+1}$

Question No: 6 (Marks: 1) - Please choose one

If $f(x) = 3x^8 + 2x + 1$ then $f'(x) =$ _____

- $3x^7 + 2$
- $24x^7 + 2$
- $3x^9 + 2x^2$
- $24x^9 + 2x^2$

Question No: 7 (Marks: 1) - Please choose one

$\frac{d(\tan x)}{dx} =$

- $\sec x$
- $\sec^2 x$
- $co\sec x$
- $co\sec^2 x$

Question No: 8 (Marks: 1) - Please choose one

If $y = \frac{1}{1-x}$ then $\frac{dy}{dx} =$

- 1
- -1
- $\frac{1}{(1-x)^2}$
- $\frac{-1}{(1-x)^2}$

Question No: 9 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- 0
- $\frac{-1}{x^2}$
- $\frac{4}{x^2}$
- $\frac{-4}{x^2}$

Question No: 10 (Marks: 1) - Please choose one

If $x^2 + y^2 = 9$ then $\frac{dy}{dx} =$

- $\frac{x}{y}$
- $\frac{-x}{y}$
- $\frac{-y}{x}$
- $\frac{y}{x}$

Question No: 11 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then
$\frac{d}{dx}\left((c)\{h(x)\}\right) =$ _____

- 0
- $\frac{d}{dx}\left(h(x)\right)$
- $\frac{d}{dx}\left(h(cx)\right)$
- $c\frac{d}{dx}\left(h(x)\right)$

Question No: 12 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$\frac{d}{dx}[f][g] =$

- $\frac{[f'][g]-[f][g']}{g^2}$
- $[f'][g']$
- $[f'][g]+[f][g']$
- $[f'][g]-[f][g']$

Question No: 13 (Marks: 1) - Please choose one

$\frac{d}{dx}[\text{cosec } x] =$ _____

$\frac{-\cos x}{1-\cos^2 x}$

- $\frac{-\cos x}{1-\cos^2 x}$
- $\frac{-\cos x}{1-\cos^2 x}$
- $\frac{1}{1-\cos^2 x}$

Question No: 14 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point g(x), then the _____is differentiable at point x .

- Composition (f o g)
- Quotient (f / g)
- Product (f . g)
- Sum (f + g)

Question No: 15 (Marks: 1) - Please choose one

$y = f\left(g\left(h(x)\right)\right)$

If

$u = g\left(h(x)\right)$

$v = h(x)$ then $\frac{dy}{dx} =$ _____

- $\frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$
- $\frac{dy\ du\ dv}{du\ dv\ dx}$
- $\frac{dv}{du} \cdot \frac{du}{dv} \cdot \frac{dy}{dx}$

Question No: 16 (Marks: 1) - Please choose one

Chain rule is a rule for differentiating _____of functions.

- Product
- Sum
- Difference
- Composition

Question No: 17 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denote points in that interval. If $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ then which of the following statement is correct?

- f is an increasing function.
- f is a decreasing function.
- f is a constant function.

Question No: 18 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then which of the following statement is correct?

- f is a decreasing function
- f is an increasing function
- f is a constant function

Question No: 19 (Marks: 1) - Please choose one

If $f''(x) > 0$ on an open interval (a,b), then which of the following statement is correct?

- f is concave up on (a, b).
- f is concave down on (a, b).
- f is linear on (a, b).

Question No: 20 (Marks: 1) - Please choose one

If $x > 0$ then $\frac{d}{dx}[\ln x] =$ _____

- 1
- x
- $\frac{1}{x}$
- $\ln \frac{1}{x}$

Question No: 21 (Marks: 1) - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

$\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$

- $\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$

Question No: 22 (Marks: 1) - Please choose one

What is the base of natural logarithm?

- 2.71
- 10
- 5
- Any real number

Question No: 23 (Marks: 1) - Please choose one

Let x_0 be critical points of the function f. Those critical points for which $f'(x_0) = 0$ are called _____of f

- Local points

► End points

► Stationary points

Question No: 24 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

► $a \log_b r$

► $r \log_b a$

► $\frac{\log_b a}{\log_b r}$

► $\log_b a + \log_b r$

Question No: 25 (Marks: 3)

Discuss the concavity of the function $f(t) = (2-t)(t-3)+3$ on any interval using second derivative test?

For all constant numbers, derivative is zero.
So,
F'(t) = 0

F''(t) = 0.....derivative doesn't exist.

Question No: 26 (Marks: 5)

Find the derivative of the function $y = \ln(1+x^3)$

$$\frac{d}{dx}[\ln(1+x^3)] = \frac{1}{(1+x^3)} \cdot \frac{dy}{dx}[1+x^3]$$
$$= \frac{1}{(1+x^3)} .3x$$
$$= \frac{3x}{(1+x^3)} ...Ans$$

Question No: 27 (Marks: 10)

Compute derivative of trigonometric function $Sin(x)$ by definition.

We want to know the derivative of $f(x) = \sin x$

so,

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)-\sin(x)+\cos(x)\sin(h)}{h}$$

$$= \lim_{h\to 0}[\sin(x)\left(\frac{\sin(h)}{h}\right)-\sin(x)\left(\frac{1-\cos(h)}{h}\right)]$$

As $\sin x$ *and* $\cos x$ *donot involve* h *so they will remain cons*$\tan t$ *here*

$$\lim_{h\to 0}\sin(x) = \sin(x)$$

$$\lim_{h\to 0}\cos(x) = \cos(x)$$

Now,

$$\frac{d}{dx}(\sin x) = \cos(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)-\sin(x)\lim_{h\to 0}\left(\frac{1-\cos(h)}{h}\right)$$

$$= \cos(x)(1)-\sin(x)(0) = \cos x$$

$$\frac{d}{dx}\sin(x) = \cos(x).....proved$$